EXAM 2 REVIEW AND GUIDED PRACTICE

Responses, feedback and frequency

This guide explains how to start a problem, choose a useful relationship, and carry the calculation through to a result you can interpret. The examples and practice questions use different systems and values from the exam. Work one topic at a time; the detailed practice solutions are collected at the end.

A useful way to study

Read a worked example with paper beside you. Cover the next line and write what you think should come next. Then compare the algebra and the reason for the step. Try the short problem at the end of each topic before reading its solution.

The six questions to keep asking

Topic	What you should be able to do
First-order response	Identify the input transform, form $Y = GR$, and use partial fractions and the Laplace table to find a step response.
Second-order response	Compare coefficients, find ω_n, ζ and ω_d, and interpret the poles.
Proportional feedback	Form the closed-loop transfer function and choose K by matching its denominator to the standard second-order form.
Sinusoidal steady state	Evaluate $G(i\omega)$, multiply the input amplitude by its magnitude, and interpret its phase.
Bode reading	Read gain and phase at one frequency and use those readings to predict the output.
Time and frequency	Compare matched curves, identify bandwidth relative to the low-frequency gain, and use the "enter and stay" settling rule.

Keep the model, input and output separate

$G(s)$ describes the system. $R(s)$ describes the applied input. Their product gives $Y(s)$ under zero initial conditions. For example, a step input contributes a factor $1/s$ to $R(s)$; it does not change the poles of the transfer function $G(s)$ itself.

In sinusoidal problems, ω is the chosen *input frequency*. It is different from the system parameter ω_n, the natural frequency. The phase ϕ is an angle, not a frequency.

Use the available formulas

The exam permits any amount of handwritten notes and a standard calculator, and includes a formula reference. This guide also ends with a compact formula page. The work you practise is choosing and applying the relationship, and explaining what the result means.

The review follows the current scope through the comparison of time-domain and frequency-domain responses.

First-order response: start with the input

Suppose a system has transfer function

$$G(s) = \frac{6}{s+3}.$$

The system starts from rest. We want to distinguish an impulse input from a step input before doing any inverse transform.

1. Identify the input transform

For the causal signals used here,

Input or time function	Transform	What it represents
$\delta(t)$	1	Unit impulse
$u_s(t)$	$1/s$	Unit step
$e^{-at}, t \geq 0$	$1/(s+a)$	Exponential

Here $u_s(t)$ denotes the unit step. Use the same table in reverse when you need the time response.

2. Form the output transform

The relationship to use is

$$Y(s) = G(s)R(s).$$

For a **unit impulse**, substitute $R(s) = 1$:

$$Y_{\text{imp}}(s) = \frac{6}{s+3}(1) \quad \Longrightarrow \quad \boxed{Y_{\text{imp}}(s) = \frac{6}{s+3}.}$$

This completes a question that asks only for the output transform.

For a **unit step**, substitute $R(s) = 1/s$ instead:

$$Y_{\text{step}}(s) = \frac{6}{s+3}\frac{1}{s} \quad \Longrightarrow \quad \boxed{Y_{\text{step}}(s) = \frac{6}{s(s+3)}.}$$

The extra factor $1/s$ comes from the input. The system has not changed; we applied a different signal to it.

3. Choose the form needed for the table

The exponential row contains one first-order denominator. Our output transform contains two factors, s and $s+3$. Split it into matching terms:

$$\frac{6}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}.$$

The unknown coefficients must be found before using the table.

Finish the step response, one line at a time

Continue with $Y(s) = 6/[s(s+3)] = A/s + B/(s+3)$.

4. Clear the denominators and compare coefficients

Multiply every term by the common denominator $s(s+3)$:

$$6 = A(s+3) + Bs = (A+B)s + 3A.$$

The left side has constant term 6 and zero coefficient of s. Therefore

$$3A = 6 \quad \Longrightarrow \quad A = 2,$$

$$A + B = 0 \quad \Longrightarrow \quad B = -2.$$

Insert those coefficients into the split expression:

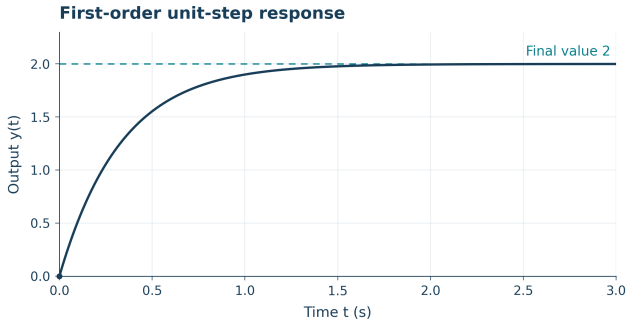
$$Y(s) = \frac{2}{s} - \frac{2}{s+3}.$$

5. Read the table in reverse

The first term corresponds to the constant 2. The second corresponds to $-2e^{-3t}$:

$$\boxed{y(t) = 2 - 2e^{-3t}, \qquad t \geq 0.}$$

At the start, $y(0) = 2 - 2 = 0$. As time increases, e^{-3t} decays to zero, so $y(t)$ approaches 2. The transient is the decaying term; the settled part is the constant 2.



Try 1: form the transform, then find the time response

A different system has $G(s) = 8/(s+4)$ and zero initial conditions.

(a) Write $Y(s)$ for a unit impulse.

(b) For a unit step, form $Y(s)$, find its partial fractions, and then find $y(t)$.

(c) Check the starting value and the value approached as t increases.

Start by writing $R(s)$ for each input. Detailed solution: Try 1 at the end.

Second-order response: read the coefficients

Consider the unit-step response of

$$T(s) = \frac{100}{s^2 + 16s + 100}.$$

We begin by comparing the denominator with the standard second-order form. This identifies the system parameters before we interpret its motion.

The standard form

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

ω_n is the natural frequency, in rad/s. ζ is the dimensionless damping ratio. For $0 < \zeta < 1$, the damped oscillation frequency is $\omega_d = \omega_n\sqrt{1 - \zeta^2}$.

1. Match the constant term

Put the two denominator polynomials beside each other:

$$s^2 + 16s + 100 \quad \leftrightarrow \quad s^2 + 2\zeta\omega_n s + \omega_n^2.$$

The constant term gives

$$\omega_n^2 = 100 \quad \Longrightarrow \quad \boxed{\omega_n = 10 \text{ rad/s.}}$$

Take the positive square root: natural frequency is positive for this model.

2. Match the coefficient of s

Use the natural frequency already found:

$$2\zeta\omega_n = 16 \quad \Longrightarrow \quad 2\zeta(10) = 16.$$

Divide by 20:

$$\boxed{\zeta = 0.8.}$$

The coefficient 16 is $2\zeta\omega_n$, not ζ by itself.

3. Find the oscillation frequency

Because $0 < 0.8 < 1$, use the underdamped relationship:

$$\omega_d = 10\sqrt{1 - 0.8^2} = 10\sqrt{0.36} = 10(0.6).$$

$$\boxed{\omega_d = 6 \text{ rad/s.}}$$

The natural frequency 10 and the oscillation frequency 6 are different quantities. The poles on the next page show how each enters the response.

The poles explain decay and oscillation

For the same denominator $s^2 + 16s + 100$, find its roots:

$$s = \frac{-16 \pm \sqrt{16^2 - 4(100)}}{2} = \frac{-16 \pm \sqrt{-144}}{2}.$$

Since $i^2 = -1$, $\sqrt{-144} = 12i$. Therefore

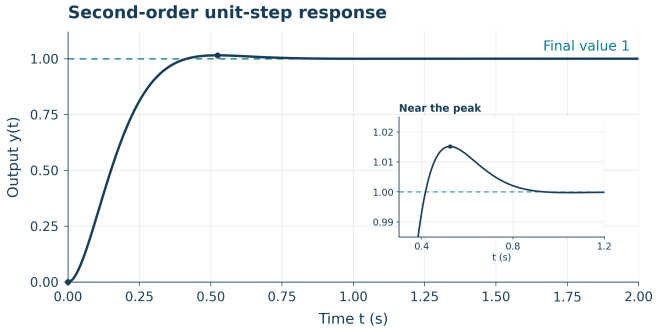
$$\boxed{p_{1,2} = -8 \pm 6i.}$$

The standard pole formula gives the same result:

$$-\zeta\omega_n \pm i\omega_d = -0.8(10) \pm 6i = -8 \pm 6i.$$

Explain the two parts separately

Pole feature	Meaning for the motion
Real part -8	Transient terms have a factor e^{-8t}, which decays. The poles are in the left half-plane, so this response is stable.
Imaginary parts ± 6	The transient includes oscillation at 6 rad/s. It can oscillate while its amplitude decreases.



Here the overshoot is small, but the imaginary part is still nonzero. Judge the decay and oscillation from the pole components, rather than requiring a large visible peak.

Try 2: parameters, poles and meaning

For

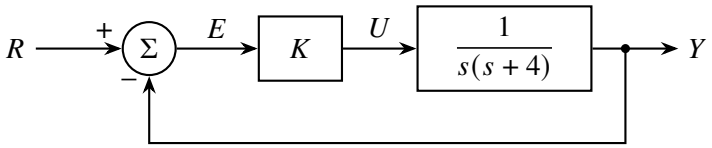
$$T(s) = \frac{36}{s^2 + 6s + 36},$$

find ω_n, ζ, ω_d, and both poles. Explain the stability using their real parts and the oscillation using their imaginary parts. You may leave square roots in exact form.

Begin with $\omega_n^2 =$ _____ and $2\zeta\omega_n =$ _____. Detailed solution: Try 2 at the end.

Unity negative feedback: follow the signals

Let the plant be $G(s) = 1/[s(s+4)]$, with controller gain $K > 0$. The output is returned unchanged, so the feedback is unity.



1. Write the two signal equations

The summing point subtracts the measured output from the reference:

$$E = R - Y.$$

The error passes through K and then G, so

$$U = KE, \qquad Y = GU = KGE.$$

The forward-path relation maps *error* to output. The closed-loop transfer function must map *reference* to output.

2. Substitute and collect the output terms

$Y = KG(R - Y)$	Substitute for E.
$Y = KGR - KGY$	Expand.
$(1 + KG)Y = KGR$	Collect the terms containing Y.

Divide by R and by $1 + KG$:

$$\boxed{T(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)}.}$$

3. Put the plant into this expression

$$T(s) = \frac{K/[s(s+4)]}{1 + K/[s(s+4)]} = \frac{K}{s(s+4) + K} = \boxed{\frac{K}{s^2 + 4s + K}}.$$

The common denominator is removed by multiplying the top and bottom by $s(s+4)$. The closed-loop characteristic equation is $s^2 + 4s + K = 0$.

Choose K by comparing coefficients

We want the closed-loop damping ratio to be $\zeta = 0.5$. Use the denominator found on the previous page:

$$s^2 + 4s + K \quad \leftrightarrow \quad s^2 + 2\zeta\omega_n s + \omega_n^2.$$

1. Write both matching relationships

$$\omega_n^2 = K, \qquad 2\zeta\omega_n = 4.$$

The constant term determines ω_n^2. The coefficient of s contains both damping and natural frequency.

2. Use the specified damping ratio

$$2(0.5)\omega_n = 4 \quad \Longrightarrow \quad \omega_n = 4 \text{ rad/s}.$$

Now use the other relationship:

$$K = \omega_n^2 = 4^2 \quad \Longrightarrow \quad \boxed{K = 16.}$$

3. Check the target in the resulting polynomial

With $K = 16$, the denominator is $s^2 + 4s + 16$. It has $\omega_n = \sqrt{16} = 4$ and

$$\zeta = \frac{4}{2(4)} = 0.5,$$

which is the requested value.

Do not identify a coefficient with the wrong parameter

The coefficient of s is $2\zeta\omega_n$, not ζ. The constant term is ω_n^2, not ω_n. Write both equalities before solving for K.

Try 3: repeat the method on another plant

A plant $G(s) = 1/[s(s + 6)]$ has proportional gain $K > 0$ and unity negative feedback. Find the closed-loop transfer function and its characteristic equation. Choose K for $\zeta = 0.5$.
Use $T = KG/(1 + KG)$, simplify its denominator, and compare it with $s^2 + 2\zeta\omega_n s + \omega_n^2$. Detailed solution: Try 3 at the end.

Sinusoidal input: find the amplitude multiplier

After the transient decays, a stable LTI system's sinusoidal output has the same frequency as the input. Its amplitude and phase can change:

$$r(t) = R\sin(\omega t) \quad \longrightarrow \quad G(s) \quad \longrightarrow \quad y_{ss}(t) = Y\sin(\omega t + \phi).$$

The two quantities to find are

$$Y = R|G(i\omega)|, \qquad \phi = \angle G(i\omega).$$

Here R and Y are amplitudes, not the transforms $R(s)$ and $Y(s)$. Also, $i^2 = -1$; engineering notation j denotes the same imaginary unit.

Worked example: identify the input before substituting

$$G(s) = \frac{6}{s+3}, \qquad r(t) = \sqrt{2}\sin(3t).$$

Compare the input with $R\sin(\omega t)$:

$$R = \sqrt{2}, \qquad \omega = 3 \text{ rad/s}.$$

The numerator 6 belongs to the system; it is not the input amplitude.

1. Evaluate at the input frequency

Substitute $s = i\omega = 3i$ into the transfer function:

$$G(3i) = \frac{6}{3+3i}.$$

Keep this complex fraction. Its magnitude and angle can be found without multiplying by the denominator's conjugate.

2. Find the magnitude

For $a + bi$, the magnitude is $\sqrt{a^2+b^2}$. In a quotient, the magnitudes divide:

$$|3+3i| = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2},$$

$$|G(3i)| = \frac{|6|}{|3+3i|} = \frac{6}{3\sqrt{2}} = \sqrt{2}.$$

3. Multiply by the input amplitude

$$Y = R|G(3i)| = \sqrt{2}\times\sqrt{2} \quad \Longrightarrow \quad \boxed{Y = 2.}$$

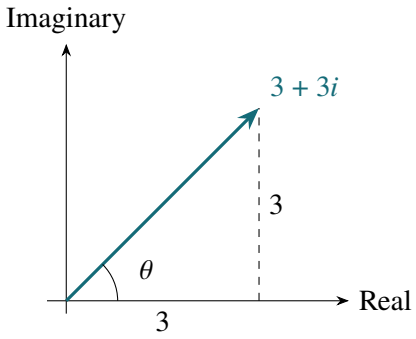
$|G(3i)|$ is a multiplier. Y is the resulting output amplitude. The gain greater than one makes this sinusoid larger; its frequency is still 3 rad/s. The next page supplies its timing shift.

Phase: use the complex angle, then write the output

For a quotient, subtract the denominator angle from the numerator angle:

$$\angle\left(\frac{z_1}{z_2}\right) = \angle z_1 - \angle z_2.$$

The positive real numerator 6 has angle zero. The denominator $3 + 3i$ lies in the first quadrant.



The triangle gives

$$\tan\theta = \frac{3}{3} = 1 \quad \Longrightarrow \quad \theta = \tan^{-1}(1) = 45^\circ = \frac{\pi}{4}.$$

Therefore

$$\boxed{\phi = 0 - \frac{\pi}{4} = -\frac{\pi}{4}.}$$

A negative phase means that the output **lags** the input.

Put amplitude, frequency and phase in the final expression

We found $Y = 2$, $\omega = 3$ rad/s and $\phi = -\pi/4$. Thus

$$\boxed{y_{ss}(t) = 2\sin\left(3t - \frac{\pi}{4}\right).}$$

Use radians inside this sine expression. In general, $\phi_{rad} = \phi_{deg}\pi/180$.

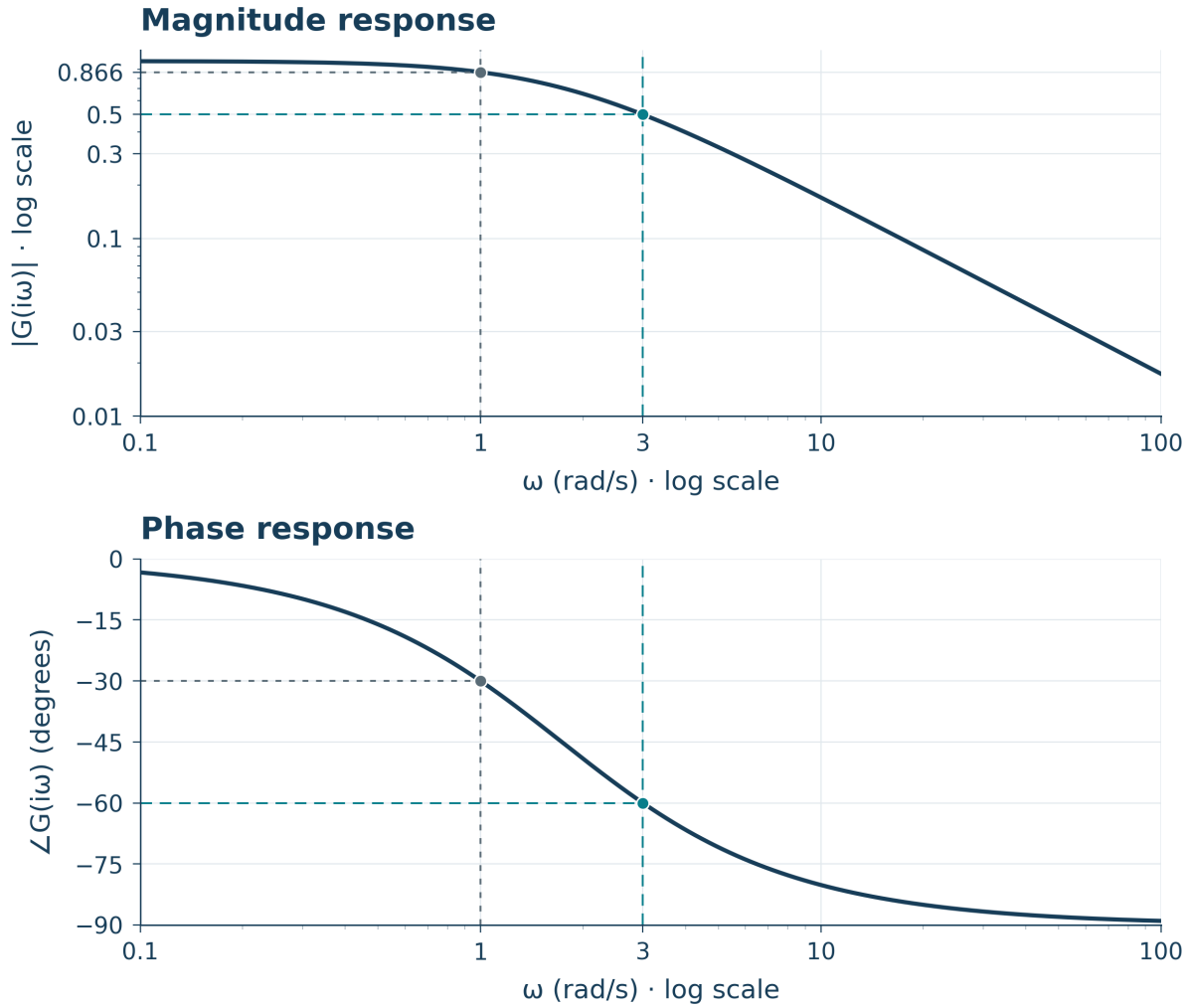
Try 4: follow the same five steps

For $G(s) = 4/(s + 4)$ and $r(t) = 2\sqrt{2}\sin(4t)$, find $G(i\omega)$, its magnitude, the output amplitude, phase and steady-state output. State whether the output leads or lags.
Start by identifying R and ω. Detailed solution: Try 4 at the end.

A Bode plot gives two readings at one frequency

The plot below describes a different system from the previous example. A Bode plot organizes the system's gain and phase over input frequency. The horizontal axis is input angular frequency ω, in rad/s. The magnitude graph gives an amplitude multiplier; the phase graph gives a timing shift in degrees.

Here the frequency and magnitude axes are logarithmic; phase is linear. Equal distances on a log axis represent equal ratios, not equal differences.



Read the same frequency on both graphs

At $\omega = 3$ rad/s, follow the teal vertical guide to each curve. Then follow the horizontal guide to its vertical axis:

$$\boxed{|G(3i)| = 0.5, \qquad \angle G(3i) = -60^\circ.}$$

The gain 0.5 means that this frequency component leaves with half its input amplitude. The negative phase means that its output lags. The curve is not a time waveform: moving right means changing the input frequency, not allowing more time to pass.

Use the Bode reading to predict the output

Use the same plotted system, with input

$$r(t) = 6\sin(3t).$$

We already read the required frequency-response values from the graph. There is no need to reconstruct a transfer function to use those readings.

1. Match the input to the marked frequency

The input has $R = 6$ and $\omega = 3$ rad/s. Use the readings at 3 rad/s, not the separate gray mark at 1 rad/s.

2. Multiply by the amplitude ratio

$$Y = R|G(3i)| = 6(0.5) \quad \Longrightarrow \quad \boxed{Y = 3.}$$

The graph's vertical value 0.5 is a ratio, not the output amplitude 3.

3. Interpret the phase and, if requested, write the output

The phase is $-60°$, so the output lags. Converting to radians,

$$-60°\frac{\pi}{180} = -\frac{\pi}{3}.$$

The full steady-state expression would be

$$\boxed{y_{ss}(t) = 3\sin\left(3t - \frac{\pi}{3}\right).}$$

Its frequency remains 3 rad/s. Changing the gain at this frequency changes amplitude; it does not replace the input frequency with the gain value.

What if the input frequency changes?

At 1 rad/s the gray guides instead show gain about 0.866 and phase $-30°$. The system is the same, but the gain and phase readings are different because the input frequency is different. Choose the frequency first, then read the two corresponding values.

Try 5: use the plotted response

For the same Bode plot, let $r(t) = 10\sin(3t)$. Read the amplitude ratio and phase at the input frequency, find the output amplitude, and state whether the output leads or lags. Give one reason from the phase plot.
If you also write $y_{ss}(t)$, convert the phase to radians. Detailed solution: Try 5 at the end.

Bandwidth: use the low-frequency value

Two different readings on a magnitude graph describe different features:

- The resonant peak M_r is the *largest magnitude*, the height of a peak if the curve has one.
- Bandwidth is a *frequency*. It is found where the gain falls to $1/\sqrt{2} \approx 0.7071$ times its low-frequency value.

For the low-pass responses here, with finite nonzero low-frequency magnitude $|G(0)|$, the bandwidth condition is

$$|G(i\omega_{\mathrm{BW}})| = \frac{|G(0)|}{\sqrt{2}}.$$

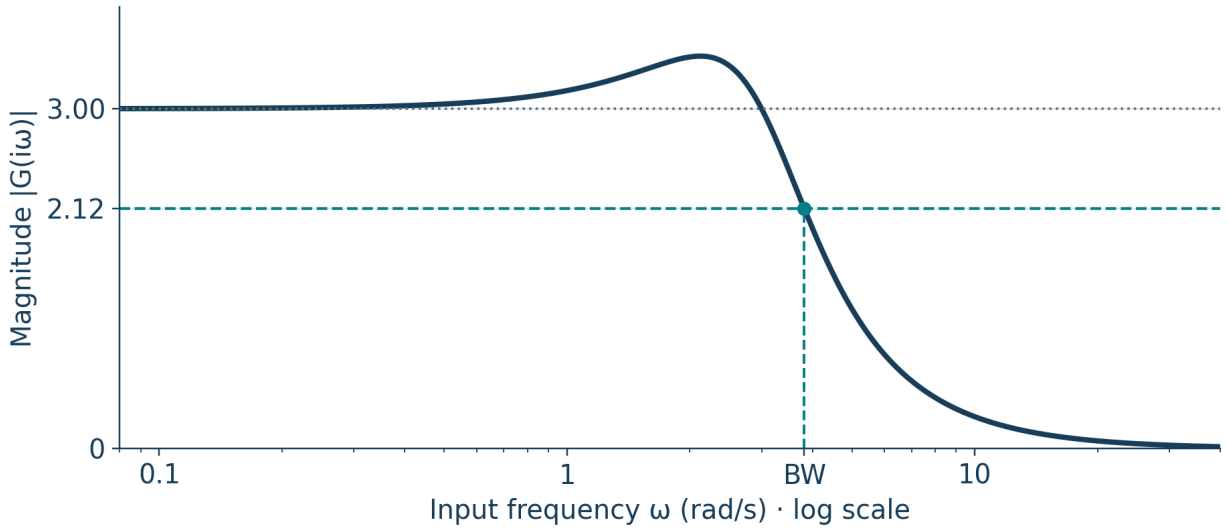
Worked example: the low-frequency gain is 3

Step 1. Read the reference level. The low-frequency plateau of the curve below is 3. This is the starting magnitude for the bandwidth calculation, even though the curve rises to a larger peak.

Step 2. Find the comparison height.

$$\frac{3}{\sqrt{2}} \approx 2.12.$$

Step 3. Use that height on the descending curve. Move horizontally from 2.12 to the falling curve, then vertically down to the frequency axis. That horizontal coordinate is the bandwidth.



The magnitude axis is linear; the frequency axis is logarithmic.

The threshold and the bandwidth are different quantities

Here 2.12 is a magnitude level. The bandwidth is the *frequency* at its descending crossing. A threshold of 0.707 applies directly only when the low-frequency magnitude is 1.

The response does not switch off at bandwidth. Its gain continues to change beyond this chosen reference point. The definition gives a consistent frequency for comparing responses.

Settling time: enter the band and remain there

A step response can enter a tolerance band, leave it during an overshoot, and return later. Settling time uses the last kind of entry: the time after which the output stays within the band.

Worked example: a final output of 5

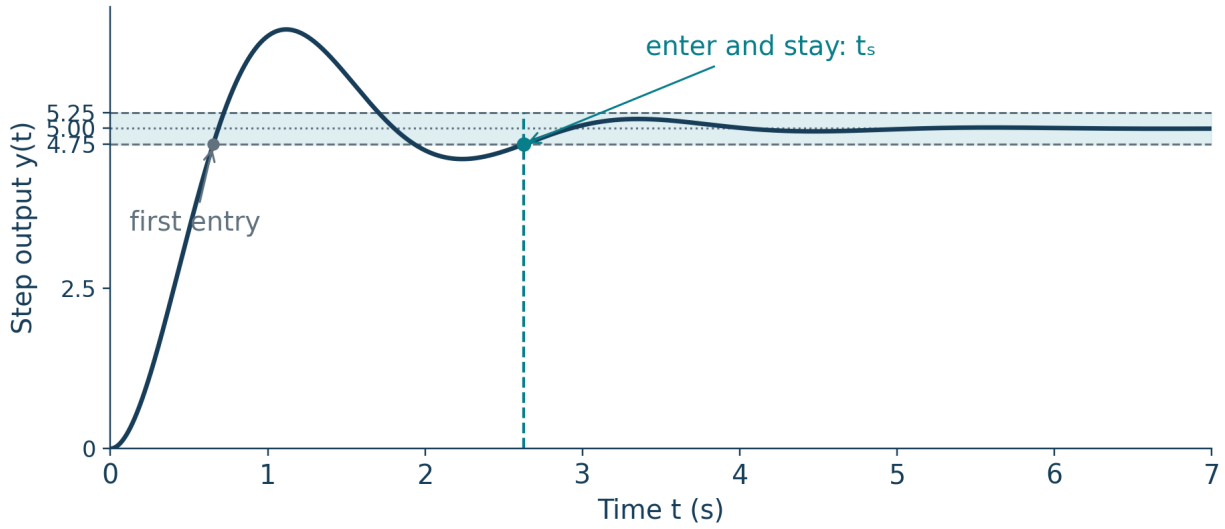
Step 1. Calculate five percent of the final value.

$$0.05(5) = 0.25.$$

Step 2. Put that tolerance on both sides of the final value.

$$5 - 0.25 \le y(t) \le 5 + 0.25 \quad \Longrightarrow \quad \boxed{4.75 \le y(t) \le 5.25.}$$

Step 3. Follow the response after an entry. The first entry below occurs before a later peak outside the shaded band. It cannot be the settling time. The marked t_s is the later entry after which the entire remaining curve stays inside.



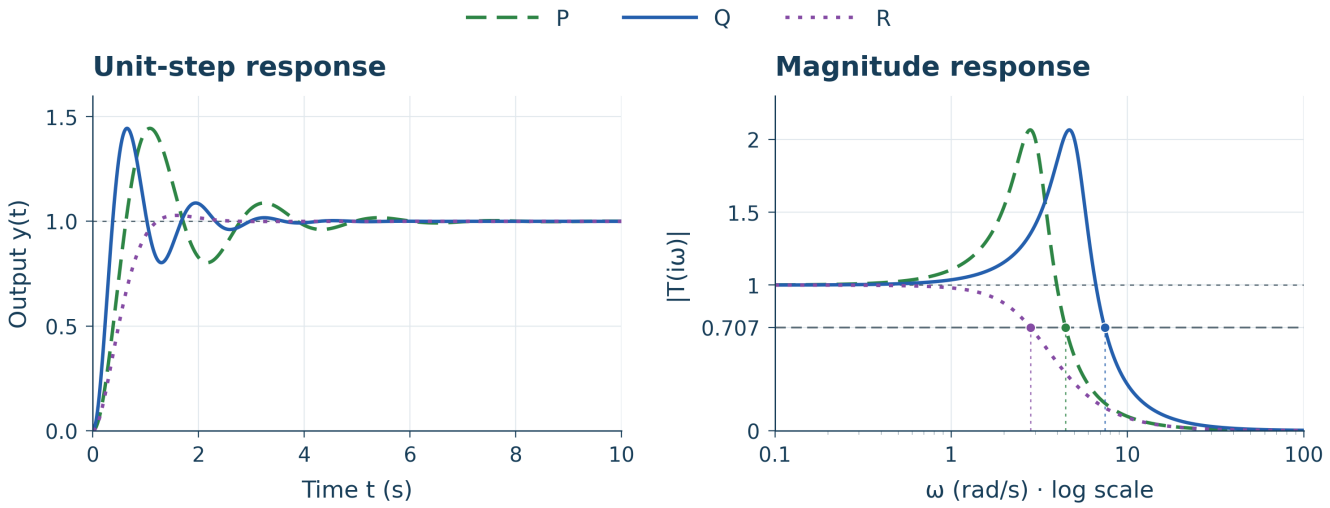
What you must check on the graph

Find the tolerance band relative to the final value. Then look forward from the proposed time: if the curve later leaves the band, that time does not yet satisfy the settling definition.

Settling is different from reaching the final value once. An oscillating curve can cross the final-value line several times before settling. The whole future response, not one crossing, decides t_s.

Compare the same systems in time and frequency

The same color and line pattern identify each system in both plots. All three are standard second-order systems with unit low-frequency gain. P and Q have the same ζ; P and R have the same ω_n. The magnitude axis is linear; the frequency axis is logarithmic.



M_p: relative step overshoot. M_r: maximum frequency-response magnitude. These are different quantities: a small step overshoot can remain even when there is no resonance hump.

Change one parameter at a time

Controlled comparison	Features to compare
Fixed ζ, larger ω_n	The time response is faster. The magnitude-response features move toward higher frequencies, with the same M_r. Step overshoot is unchanged.
Fixed ω_n, larger ζ	Step overshoot is smaller in the underdamped range. M_r decreases toward 1. For $\zeta \geq 1/\sqrt{2}$, there is no nonzero-frequency resonance hump: the maximum is 1 at DC. Bandwidth is narrower.

Use one visible feature from *each* graph when connecting the two domains. Do not infer a universal rule that increasing damping always decreases settling time: the "enter and stay" test still applies.

Try 6: specifications and matched graphs

(a) A magnitude plot has low-frequency value 5. What magnitude level must you use to find its bandwidth?

(b) A step response settles to 8. Find its ±5% band. It first enters the band, later reaches 8.7, then crosses the upper band edge on its way down and remains inside. Which entry satisfies the settling definition? Explain.

(c) In the P/Q/R plots, which of P and Q has larger ω_n? Which of P and R has larger ζ? Which system has largest bandwidth? Support each choice using the indicated curves.

Detailed solution: Try 6 at the end.

Formula guide: choose, substitute, interpret

Use the formulas with the steps shown in the worked examples. i and j both denote the imaginary unit.

Inputs and output transforms

$$\mathcal{L}\{\delta(t)\} = 1, \qquad \mathcal{L}\{u_s(t)\} = \frac{1}{s}, \qquad \mathcal{L}\{e^{-at}\} = \frac{1}{s+a}.$$

For zero initial conditions, $Y(s) = G(s)R(s)$. Transform a sum or invert a sum term by term, keeping the coefficients.

$$\frac{C}{s(s+a)} = \frac{A}{s} + \frac{B}{s+a} \quad \Longrightarrow \quad C = A(s+a) + Bs.$$

Second-order parameters and poles

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \qquad \omega_n^2 = c, \quad 2\zeta\omega_n = b \quad \text{for } s^2 + bs + c.$$

For $0 < \zeta < 1$,

$$\omega_d = \omega_n\sqrt{1-\zeta^2}, \qquad p_{1,2} = -\zeta\omega_n \pm i\omega_d.$$

The quadratic formula is $s = [-b \pm \sqrt{b^2 - 4ac}]/(2a)$ for $as^2 + bs + c = 0$. Negative pole real parts give decaying transients; nonzero imaginary parts give oscillation.

Unity negative feedback

$$E = R - Y, \qquad Y = KGE, \qquad T(s) = \frac{KG(s)}{1 + KG(s)}.$$

If $G = N/D$, then $T = KN/(D + KN)$. Use the denominator $D + KN$ for the closed-loop characteristic equation.

Complex numbers and sinusoidal response

$$|a + bi| = \sqrt{a^2 + b^2}, \quad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \quad \angle\frac{z_1}{z_2} = \angle z_1 - \angle z_2.$$

For a positive real part, $\angle(a + bi) = \tan^{-1}(b/a)$. A positive real number has angle zero.

$$r(t) = R\sin(\omega t), \quad Y = R|G(i\omega)|, \quad \phi = \angle G(i\omega), \quad y_{\text{ss}}(t) = Y\sin(\omega t + \phi).$$

$$\phi_{\text{rad}} = \phi_{\text{deg}}\frac{\pi}{180}, \qquad 45^\circ = \frac{\pi}{4}, \quad 60^\circ = \frac{\pi}{3}.$$

Here R, Y are amplitudes, not $R(s), Y(s)$. This output is after the transient decays. The frequency stays the same; negative phase means lag.

Bandwidth and the settling band

For the low-pass responses here, $G(0)$ is finite and nonzero:

$$|G(i\omega_{\text{BW}})| = \frac{|G(0)|}{\sqrt{2}} \approx 0.7071|G(0)| \qquad \text{on the descending curve.}$$

$$|y(t) - y_\infty| \le 0.05|y_\infty|.$$

Here y_∞ is the final step-response value. Settling time is the first time after which this stays true. Keep the other parameter fixed and compare the same labeled curves in both graphs.

Practice solutions: first- and second-order response

Try 1: start by distinguishing the inputs

(a) Unit impulse. $R(s) = 1$, so

$$\boxed{Y_{\text{imp}}(s) = G(s)R(s) = \frac{8}{s+4}.}$$

(b) Unit step. Now $R(s) = 1/s$, giving

$$Y_{\text{step}}(s) = \frac{8}{s(s+4)}.$$

Set $8/[s(s+4)] = A/s + B/(s+4)$ and multiply through:

$$8 = A(s+4) + Bs = (A+B)s + 4A.$$

Thus $4A = 8$ gives $A = 2$, and $A + B = 0$ gives $B = -2$.

$$Y_{\text{step}}(s) = \frac{2}{s} - \frac{2}{s+4} \quad \Longrightarrow \quad \boxed{y(t) = 2 - 2e^{-4t}, \quad t \geq 0.}$$

(c) Check. $y(0) = 2 - 2 = 0$; as t increases the exponential decays and $y(t)$ approaches 2. The check supports both the minus sign and the constant term in the result.

Try 2: coefficients first, then poles

Compare $s^2 + 6s + 36$ with $s^2 + 2\zeta\omega_n s + \omega_n^2$:

$$\omega_n^2 = 36 \Rightarrow \omega_n = 6 \text{ rad/s}, \qquad 2\zeta(6) = 6 \Rightarrow \zeta = 0.5.$$

Then

$$\omega_d = 6\sqrt{1 - 0.5^2} = 6\sqrt{3/4} = 3\sqrt{3} \text{ rad/s}.$$

The poles are

$$p = -\zeta\omega_n \pm i\omega_d = \boxed{-3 \pm 3\sqrt{3}\,i}.$$

Their negative real part produces a decaying factor e^{-3t}, so the system is stable. The imaginary parts are nonzero, so the transient oscillates at $3\sqrt{3} \approx 5.20$ rad/s. Stability and oscillation can occur together because the oscillation amplitude decreases.

Practice solutions: feedback and sinusoidal response

Try 3: derive the closed loop before choosing K

For $G = 1/[s(s+6)]$,

$$T = \frac{KG}{1+KG} = \frac{K}{s(s+6)+K} = \boxed{\frac{K}{s^2+6s+K}}.$$

The characteristic equation is $s^2 + 6s + K = 0$. Comparing its coefficients with the standard second-order polynomial gives

$$\omega_n^2 = K, \qquad 2\zeta\omega_n = 6.$$

Use the requested $\zeta = 0.5$:

$$2(0.5)\omega_n = 6 \Rightarrow \omega_n = 6 \text{ rad/s} \Rightarrow \boxed{K = 36.}$$

Substituting back yields $2\zeta\omega_n = 2(0.5)(6) = 6$, as required.

Try 4: evaluate only at the input frequency

The input $2\sqrt{2}\sin(4t)$ has $R = 2\sqrt{2}$ and $\omega = 4$ rad/s. Substitute $s = 4i$:

$$G(4i) = \frac{4}{4+4i}, \qquad |4+4i| = \sqrt{4^2+4^2} = 4\sqrt{2}.$$

Therefore

$$|G(4i)| = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}, \qquad Y = 2\sqrt{2}\frac{1}{\sqrt{2}} = \boxed{2}.$$

The numerator angle is zero. The denominator has angle $\tan^{-1}(4/4) = \pi/4$, so

$$\phi = 0 - \frac{\pi}{4} = -\frac{\pi}{4}.$$

The output lags because its phase shift is negative. Its steady-state output is

$$\boxed{y_{ss}(t) = 2\sin\left(4t - \frac{\pi}{4}\right).}$$

The amplitude is 2, the angular frequency is 4 rad/s, and the phase shift is $-45°$. Those three readings refer to different features of the same sinusoid.

Practice solutions: read and compare the graphs

Try 5: a graph gives the multiplier and phase

The input $10\sin(3t)$ has amplitude 10 and frequency 3 rad/s. At that frequency the graph gives $|G(3i)| = 0.5$ and $\angle G(3i) = -60°$.

$$Y = 10(0.5) = \boxed{5}.$$

The negative phase means lag. If writing the full output, first convert $-60°$ to $-\pi/3$:

$$\boxed{y_{ss}(t) = 5\sin\left(3t - \frac{\pi}{3}\right).}$$

The input changed from the worked example, but the system and chosen frequency did not, so the gain and phase readings stay the same.

Try 6: apply each definition to the correct reference

(a) Bandwidth level. Multiply the low-frequency gain by $1/\sqrt{2}$:

$$\boxed{\frac{5}{\sqrt{2}} \approx 3.54.}$$

Find the descending crossing at this magnitude, then read its frequency. The value 3.54 is the required height, not the bandwidth itself.

(b) Settling band. Five percent of 8 is $0.05(8) = 0.4$:

$$\boxed{7.6 \le y(t) \le 8.4.}$$

The first entry does not qualify because the later peak 8.7 exceeds 8.4. The later downward crossing of 8.4 is the settling entry under the stated condition that the response then stays in the band.

(c) Matched systems. Q has larger ω_n than P: their damping is equal, but Q responds faster and its frequency features occur farther to the right. R has larger ζ than P: their natural frequencies are equal, but R has much less step overshoot and a smaller maximum frequency-response magnitude. R has no nonzero-frequency resonance hump, yet still has a small step overshoot. Q has the largest bandwidth because its descending crossing of the 0.707 level is farthest right.

The parameters used to draw these practice curves are

System	ω_n (rad/s)	ζ
P	3	0.25
Q	5	0.25
R	3	0.75

The graph comparisons do not require calculating a resonant peak or an exact bandwidth from these parameters.