

# FEJ2: A Consistent Visual-Inertial State Estimator Design

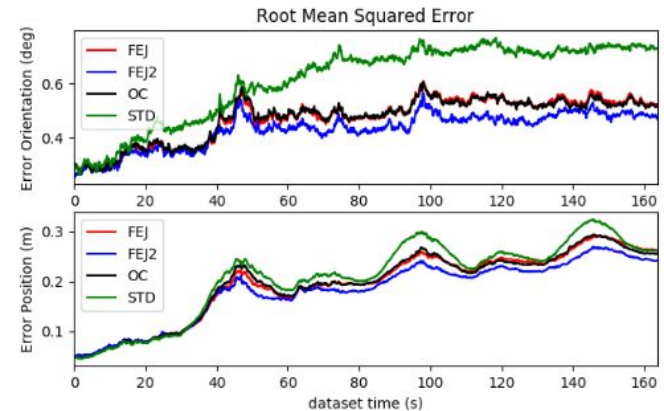
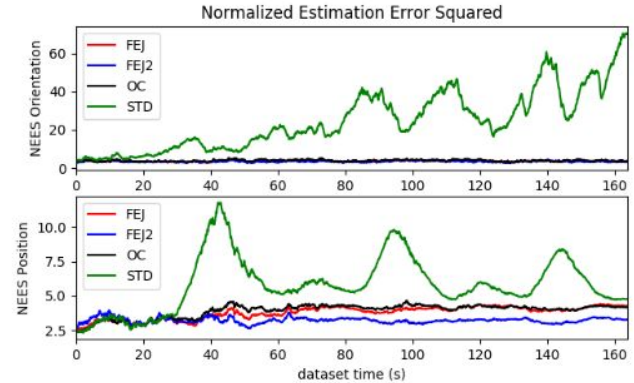
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# Introduction

- Filter-based visual-inertial estimators
  - 4 d.o.f unobservable ideally (yaw + pos.)
  - linearizing at current state estimates causes information gains in unobs.
  - Covariance becomes overconfident (inconsistent)
- First-estimates Jacobian (FEJ)
  - Fixes Jacobians at first estimates to enforce 4 d.o.f (consistent)
  - Fixes Jacobians introduce unmodelled errors
- We propose **FEJ2**
  - Addresses the unmodelled errors of FEJ
  - Shown to improve performance

↙ *NEES is large since covariance is overconfident*



[1] Castellanos, José A., José Neira, and Juan D. Tardós. "Limits to the consistency of EKF-based SLAM." IFAC Proceedings Volumes 37.8 (2004): 716-721.

[2] Hesch, Joel A., et al. "Camera-IMU-based localization: Observability analysis and consistency improvement." The International Journal of Robotics Research 33.1 (2014): 182-201.

[3] Huang, Guoquan P., Anastasios I. Mourikis, and Stergios I. Roumeliotis. "Observability-based rules for designing consistent EKF SLAM estimators." The International Journal of Robotics Research 29.5 (2010): 502-528.

nonlinear measurement function

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{n}$$

$\uparrow$  state vector  
 $\rightarrow$  noise

sensor measurement

$$\begin{aligned}
 &= \mathbf{h}(\hat{\mathbf{x}}) + (\bar{\mathbf{H}} + \hat{\mathbf{H}} - \bar{\mathbf{H}})\tilde{\mathbf{x}} + \mathbf{n} \\
 &= \mathbf{h}(\hat{\mathbf{x}}) + \bar{\mathbf{H}}\tilde{\mathbf{x}} + (\hat{\mathbf{H}} - \bar{\mathbf{H}})\tilde{\mathbf{x}} + \mathbf{n} \\
 &= \mathbf{h}(\hat{\mathbf{x}}) + \bar{\mathbf{H}}\tilde{\mathbf{x}} + \Delta\mathbf{H}\tilde{\mathbf{x}} + \mathbf{n}
 \end{aligned}$$

$$\hat{\mathbf{r}} = \mathbf{z} - \mathbf{h}(\hat{\mathbf{x}}) \simeq \bar{\mathbf{H}}\tilde{\mathbf{x}} + \Delta\mathbf{H}\tilde{\mathbf{x}} + \mathbf{n}$$

$$\Delta\mathbf{U}^\top \hat{\mathbf{r}} = \Delta\mathbf{U}^\top \bar{\mathbf{H}}\tilde{\mathbf{x}} + \Delta\mathbf{U}^\top \Delta\mathbf{H}\tilde{\mathbf{x}} + \Delta\mathbf{U}^\top \mathbf{n}$$

$$\Rightarrow \mathbf{r}^* = \mathbf{H}^* \tilde{\mathbf{x}} + \mathbf{n}^*$$

### FEJ

- Evaluate the measurement Jacobian at the first state estimate
- Assumes  $\Delta\mathbf{H}$  is **zero** to improve consistency
- Introduce unmodelled errors

### FEJ2

- $\Delta\mathbf{H} = \hat{\mathbf{H}} - \bar{\mathbf{H}}$  captures linearization point changes between the **first** and **best** state estimates
- Project onto the nullspace of  $\Delta\mathbf{H}$  to remove
- Keeps the **correct** unobservable subspace
- Better consistency than FEJ

# Results and Conclusion

- Simulate inertial and bearing measurements under different VINS frameworks
- Monocular and stereo measurements
- Different measurement noise

Noise (pixel)	Est.	RMSE Ori. (deg)		RMSE Pos. (m)		NEES Ori.		NEES Pos.	
		mono / stereo	mono / stereo	mono / stereo	mono / stereo	mono / stereo	mono / stereo		
1	STD	0.412 / 0.344	0.130 / 0.109	23.874 / 15.447	4.911 / 4.874				
	OC	0.242 / 0.257	0.119 / 0.100	3.290 / 3.599	3.540 / 3.416				
	FEJ	0.242 / 0.256	0.120 / 0.100	3.284 / 3.438	3.617 / 3.322				
	FEJ2	<b>0.238 / 0.238</b>	<b>0.118 / 0.095</b>	<b>3.150 / 3.324</b>	<b>3.443 / 2.965</b>				
3	STD	2.139 / 0.888	0.402 / 0.310	407.221 / 33.852	13.212 / 7.235				
	OC	0.716 / 0.723	0.301 / 0.300	3.964 / 4.395	5.051 / 4.839				
	FEJ	0.861 / 0.704	0.289 / 0.298	4.965 / 4.163	4.763 / 4.656				
	FEJ2	<b>0.650 / 0.663</b>	<b>0.264 / 0.277</b>	<b>3.198 / 3.790</b>	<b>3.581 / 3.636</b>				

**FEJ2 achieves better consistency and accuracy!**

## Summary

- Develop a novel **consistent** estimator design for VINS
- FEJ2 **accurately** models linearization errors of FEJ
- Theoretical proofs, simulations and real-world experiments show FEJ2 achieves **better performance**

**Thank you!**  
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